Landau–Drude diamagnetism: fluctuation, dissipation and decoherence

2006 J. Phys.: Condens. Matter 18 10029

(http://iopscience.iop.org/0953-8984/18/44/004)

J. Phys.: Condens. Matter **18** (2006) 10029–10041 doi:10.1088/0953-8984/18/44/004

Landau–Drude diamagnetism: fluctuation, dissipation and decoherence

Malay Bandyopadhyay[1] **and Sushanta Dattagupta**[1,2,3]

[1] S N Bose National Centre for Basic Sciences, JD Block, Sector III, Salt Lake, Kolkata 700098, India
[2] Indian Institute of Science Education and Research, HC-VII, Salt Lake, Kolkata-700106, India
[3] Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore 560064, India

E-mail: malay@bose.res.in and sdgupta@bose.res.in

Received 4 August 2006, in final form 26 September 2006
Published 20 October 2006
Online at stacks.iop.org/JPhysCM/18/10029

Abstract

Starting from a quantum Langevin equation (QLE) of a charged particle coupled to a heat bath in the presence of an external magnetic field, we present a fully dynamical calculation of the susceptibility tensor. In a different 'equilibrium approach', we further evaluate the position autocorrelation function by using the Gibbs ensemble. This quantity is shown to be related to the imaginary part of the dynamical susceptibility, thereby validating the fluctuation–dissipation theorem in the context of dissipative diamagnetism. Finally, we present an overview of coherence-to-decoherence transition in the realm of dissipative diamagnetism at zero temperature. The analysis underscores the importance of the details of the relevant physical quantity, as far as coherence-to-decoherence transition is concerned.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The problem of a quantum charged particle in the presence of a magnetic field is an old and important one [1]. When Landau gave the theory of diamagnetism, a major breakthrough in solid-state physics was made possible [2, 3]. The physics of Landau levels is of great interest in the quantum Hall effect [4] and high-temperature superconductivity [5]. In the present paper we address the issue of what happens when we combine the Landau problem with the Drude transport treatment, which naturally brings in the phenomenon of environment-induced dynamics [6].

The consequences of coupling of a system to its environment are threefold. First, energy may be transferred irreversibly from the system to the environment in the manner of dissipation [7–9]. Second, the spontaneous fluctuations in systems in thermodynamic equilibrium, maintained by its coupling to the environment, govern the response of the system degrees of freedom to weak, external stimuli [10, 11]. Finally, the entanglement between the system and the environment degrees of freedom destroys the coherent superposition of quantum states, leading to decoherence [12].

We discuss all three above-mentioned effects in the context of Landau diamagnetism, which is inherently and intrinsically quantum in nature. For the purpose of investigating fluctuation, dissipation and decoherence in what we call Landau–Drude diamagnetism [13], it is convenient to use the formulation given by Ford *et al* [14, 15], following the classical treatment due to Zwanzig [16]. Starting from the Feynman–Vernon model, in which a particle moving in an arbitrary potential is assumed to be linearly coupled to a collection of quantum harmonic oscillators [17], these authors derived a quantum Langevin equation (QLE). We use this QLE as the basis of our further discussion, in what may be referred to as the Einstein approach to statistical physics [18].

At this stage it is important to indicate in what ways our present work is an advancement on existing results in the literature, in order to put matters into perspective. Ford *et al* [19] had solved the problem of a charged oscillator in a harmonic potential well and linearly coupled to a heat bath using the generalized QLE. This solution was further extended by Li *et al* [15], but in the presence of a magnetic field. From the asymptotic expression, which is obtained in the limit of time t approaching infinity, these authors derived the influence of dissipation on the diamagnetic moment. While the diamagnetism is the first moment of an underlying quantum distribution function, we go beyond this in the present paper by treating the fluctuations in the asymptotic state, embodied in the generalized susceptibility tensor. We further connect the latter, derived from a 'nonequilibrium' QLE approach, to the position autocorrelation function calculated from the 'equilibrium' Gibbsian ensemble form of the Euclidean action for the Feynmann–Vernon model. This connection allows us to establish a relation between the position autocorrelation function and the imaginary part of the susceptibility—a statement of the fluctuation–dissipation theorem—and thus unify equilibrium and nonequilibrium statistical mechanics, in the context of dissipative diamagnetism. This is a new result.

The destruction of quantum coherence by environment-induced dissipation is of central interest in atomic physics [20], condensed matter physics [21], as well as chemical and biological reactions [22]. We discuss this environment-induced decoherence in the context of dissipative diamagnetism. Landau diamagnetism has its origin in coherent circular motion of the electron in a plane normal to the magnetic field. This coherent motion is disturbed due to interaction with environmental degrees of freedom, e.g. defects, phonons, etc. We illustrate how the system transitions from the coherent 'Landau regime' to the decoherent 'Bohr-Van Leeuwen regime' [23, 24]. Egger *et al* [25] discussed the environment-induced destruction of quantum coherence for the damped harmonic oscillator and for the dissipative two-state system and have established the dependence of this transition on the initial state of preparation. Here we have extended this study of Egger *et al* [25] and have shown that the coherent–decoherent transition depends on the particular dynamical quantity (e.g. correlation function, occupation probability, etc) under consideration for the case of Landau–Drude diamagnetism too.

This paper is organized as follows. In section 2 we discuss our model Hamiltonian and the corresponding QLE. In section 3 we calculate the generalized susceptibility tensor. Section 4 deals with the position autocorrelation function and its relation to the susceptibility, thus establishing the fluctuation–dissipation theorem in the context of dissipative diamagnetism. In section 5 we study the coherence-to-decoherence transition. Finally, we summarize our results and present a few concluding remarks in section 6.

2. Model, QLE and Einstein approach

We start with the Feynman–Vernon Hamiltonian for a charged particle in a magnetic field, $\vec{B}$, coupled to an environment of quantum harmonic oscillators [17]. In order to incorporate the contribution of the boundary electrons, we introduce a confining harmonic trap, described by the second term on the right-hand side of equation (1) below. The effect of this term can be removed at the end of the calculation by setting $\omega_0 = 0$. This trick is due originally to Darwin in the context of the equilibrium partition function [26]. Thus the underlying Hamiltonian can be written as

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + \frac{1}{2}m\omega_0^2\vec{q}^{\,2} + \sum_j \left[\frac{1}{2m_j}\vec{p}_j^{\,2} + \frac{1}{2}m_j\omega_j^2(\vec{q}_j - \vec{q})^2\right], \quad (1)$$

where $\vec{p}$ and $\vec{q}$ are the momentum and position operators of the particle, and $\vec{A}$ is the vector potential. Now, following Ford *et al* [14, 15], one can write the QLE emanating from equation (1) as [13]

$$m\ddot{\vec{q}} + \int_{-\infty}^{t} \mathrm{d}t'\gamma(t - t')\dot{\vec{q}}(t') + m\omega_0^2\vec{q} - \frac{e}{c}(\dot{\vec{q}} \times \vec{B}) = \vec{F}(t), \quad (2)$$

where the auto-correlation and the commutator of $\vec{F}(t)$ are given by

$$\langle\{F_\alpha(t), F_\beta(t')\}\rangle = \delta_{\alpha\beta}\frac{2}{\pi}\int_0^\infty \mathrm{Re}[\tilde{\gamma}(\omega + \mathrm{i}0^+)]\hbar\omega \coth\left(\frac{\hbar\omega}{2k_\mathrm{B}T}\right)\cos\{\omega(t - t')\}\,\mathrm{d}\omega, \quad (3)$$

$$\langle[F_\alpha(t), F_\beta(t')]\rangle = \delta_{\alpha\beta}\frac{2}{\mathrm{i}\pi}\int_0^\infty \mathrm{Re}[\tilde{\gamma}(\omega + \mathrm{i}0^+)]\hbar\omega \sin\{\omega(t - t')\}\,\mathrm{d}\omega, \quad (4)$$

where $\tilde{\gamma}(s) = \int_0^\infty \mathrm{d}t \exp(\mathrm{i}st)\gamma(t)$ $(\mathrm{Im}\, s > 0)$.

At this stage we introduce the nomenclature of Ohmic dissipation as well as non-Ohmic dissipation. Defining the spectral density of the environmental degrees of freedom as $J(\omega) = \frac{\pi}{2}\sum_{j=1}^{N} m_j\omega_j^3\delta(\omega - \omega_j)$, we can rewrite the memory kernel $\gamma(t)$ in terms of the spectral density as

$$\gamma(t) = \Theta(t)\frac{2}{m\pi}\int_0^\infty \mathrm{d}\omega\,\frac{J(\omega)}{\omega}\cos(\omega t), \quad (5)$$

where $\Theta(t)$ is the Heaviside step function. Often in condensed-matter physics, we deal with physical situations which can be described by such a Caldeira–Leggett model, as given in equation (1), consisting of only one or a few relevant dynamical variables in contact with a huge environment which is assumed to be a collection of harmonic oscillators [9, 27, 28]. It has been shown by Chang and Chakravarty that a Fermionic heat bath comprising electron–hole excitations near the Fermi surface, as appropriate for a metal, can indeed be represented by Bosonic operators, which are just the second quantized form of the harmonic oscillator variables of the Caldeira–Leggett model, especially when Ohmic dissipation is assumed [29]. In the Ohmic case, damping is frequency-independent and the spectral density $J(\omega) = m\gamma\omega$. The memory kernel $\gamma(t - t')$ is thus replaced by $m\gamma\delta(t - t')$, so that $\mathrm{Re}[\tilde{\gamma}(\omega + \mathrm{i}0^+)]$ reduces to $m\gamma$, a constant. In this limit we get an ordinary Langevin equation. It is interesting to note that the underlying stochastic process is still non-Markovian, even though there is no memory. On the other hand, the non-Ohmic case can be realized when the bath consists of phonons, as appropriate, for instance, in the tunnelling of an atom in the bulk [9]. Recently, Louis and Sethna [30] have shown that the case of tunnelling between surfaces corresponds to 'Ohmic' dissipation, in contrast to the bulk case, where the dissipation is of the 'super-Ohmic' variety. In the non-Ohmic case, for a bath comprising acoustic phonons, the spectral density is defined as $J(\omega) = m\tilde{\gamma}(\omega)$, where $\tilde{\gamma}(\omega) = \gamma\omega^3$ [31]. The damping kernel $\tilde{\gamma}(\omega)$ then brings in memory-friction effects.

3. Generalized susceptibility tensor

In this section we consider the linear response of the position coordinate to an external force $\vec{f}(t)$, assumed to be small. By imagining the force to have been switched on at time $t = -\infty$, all transient effects can be ignored and the non-transient response can be captured by the frequency-dependent generalized susceptibility. The corresponding QLE now reads

$$m\ddot{\vec{q}} + \int_{-\infty}^{t} dt'\gamma(t-t')\dot{\vec{q}}(t') + m\omega_0^2\vec{q} - \frac{e}{c}(\dot{\vec{q}} \times \vec{B}) = \vec{F}(t) + \vec{f}(t). \tag{6}$$

Introducing

$$\tilde{Z}_i(\omega) = \int_0^{\infty} dt e^{i\omega t} Z_i(t) \qquad (i = 1, 2, 3, 4;\ Z_1 = \gamma,\ Z_2 = q_\beta,\ Z_3 = F_\alpha,\ Z_4 = f_\alpha), \tag{7}$$

where $\epsilon_{\alpha\beta\rho}$ is the Levi-Civita symbol, and α, β, ρ are the three spatial directions (i.e. $\alpha, \beta, \rho = x, y, z$), we can rewrite equation (6) in a Fourier-transformed form:

$$\left[(m(\omega_0^2 - \omega^2) - i\omega\tilde{\gamma}(\omega))\delta_{\alpha\beta} + i\omega\frac{e}{c}\epsilon_{\alpha\beta\rho}B_\rho\right]\tilde{q}_\beta(\omega) = \tilde{F}_\alpha(\omega) + \tilde{f}_\alpha(\omega). \tag{8}$$

Equation (8) can be recast as the inverse of equation (6) in the Fourier space:

$$Y_{\alpha\beta}(\omega)\tilde{q}_\beta(\omega) = [\tilde{F}_\alpha(\omega) + \tilde{f}_\alpha(\omega)], \tag{9}$$

with

$$Y(\omega) = \begin{pmatrix} \Delta(\omega) & i\omega\frac{e}{c}B_z & -i\omega\frac{e}{c}B_y \\ -i\omega\frac{e}{c}B_z & \Delta(\omega) & i\omega\frac{e}{c}B_x \\ i\omega\frac{e}{c}B_y & -i\omega\frac{e}{c}B_x & \Delta(\omega) \end{pmatrix}, \tag{10}$$

where $\Delta(\omega) = m(\omega_0^2 - \omega^2) - i\omega\tilde{\gamma}(\omega)$. From linear response theory, one can write [32]

$$q_\alpha(t) = \int_{-\infty}^{t} ds\, \chi_{\alpha\beta}(t-s)(F_\beta(s) + f_\beta(s)), \tag{11}$$

where $\chi_{\alpha\beta}$ is the generalized susceptibility tensor. In Fourier-transformed form, equation (11) becomes

$$\tilde{q}_\alpha(\omega) = \chi_{\alpha\beta}(\omega)[\tilde{F}_\beta(\omega) + \tilde{f}_\beta(\omega)]. \tag{12}$$

Comparing equation (12) with equation (9), the generalized susceptibility can be evaluated from the following equation:

$$\chi_{\alpha\beta} = [Y^{-1}(\omega)]_{\alpha\beta}. \tag{13}$$

Clearly

$$\chi(\omega) = \frac{1}{\det[Y(\omega)]}\begin{pmatrix} \chi_{xx} & \chi_{xy} & \chi_{xz} \\ \chi_{yx} & \chi_{yy} & \chi_{yz} \\ \chi_{zx} & \chi_{zy} & \chi_{zz} \end{pmatrix}, \tag{14}$$

where

$$\begin{aligned} \det[Y(\omega)] &= \Delta(\omega)\left[\Delta^2(\omega) - \left(\omega\frac{e}{c}\right)^2\vec{B}^2\right]; \\ \chi_{ii} &= \Delta^2(\omega) - \left(\omega\frac{e}{c}\right)^2 B_i^2, \qquad (i = x, y, z); \\ \chi_{xy} &= \chi_{yx}^* = -\left(\omega\frac{e}{c}\right)^2 B_x B_y - i\omega\frac{e}{c}B_z\Delta(\omega); \\ \chi_{xz} &= \chi_{zx}^* = -\left(\omega\frac{e}{c}\right)^2 B_x B_z + i\omega\frac{e}{c}B_y\Delta(\omega); \\ \chi_{yz} &= \chi_{zy}^* = -\left(\omega\frac{e}{c}\right)^2 B_y B_z - i\omega\frac{e}{c}B_\alpha\Delta(\omega), \end{aligned} \tag{15}$$

where $(*)$ denotes the complex conjugate of the corresponding variable. The expression is simplified when the magnetic field is taken along the z-axis, thus

$$\chi(\omega) = \frac{1}{\det[Y(\omega)]} \begin{pmatrix} \Delta^2(\omega) & -\mathrm{i}\omega\frac{e}{c}\Delta(\omega)B & 0 \\ \mathrm{i}\omega\frac{e}{c}\Delta(\omega)B & \Delta^2(\omega) & 0 \\ 0 & 0 & \Delta^2(\omega) - (\omega\frac{e}{c})^2 B^2 \end{pmatrix}. \tag{16}$$

For this particular case, the real part of the susceptibility is

$$\chi'_{xx} = \chi'_{yy} = \frac{1}{2m^2}\left[\frac{(\omega_0^2 - \omega^2 + \omega\omega_\mathrm{c}/2)}{(\omega^2 - \omega_0^2 + \omega\omega_\mathrm{c})^2 + \frac{\omega^2\tilde{\gamma}^2(\omega)}{m^2}} + \frac{(\omega_0^2 - \omega^2 - \omega\omega_\mathrm{c}/2)}{(\omega^2 - \omega_0^2 - \omega\omega_\mathrm{c})^2 + \frac{\omega^2\tilde{\gamma}^2(\omega)}{m^2}}\right], \tag{17}$$

and the imaginary part is

$$\chi''_{xx} = \chi''_{yy} = \frac{\tilde{\gamma}(\omega)\omega}{2m^2}\left[\frac{1}{(\omega^2 - \omega_0^2 + \omega\omega_\mathrm{c})^2 + \frac{\omega^2\tilde{\gamma}^2(\omega)}{m^2}} + \frac{1}{(\omega^2 - \omega_0^2 - \omega\omega_\mathrm{c})^2 + \frac{\omega^2\tilde{\gamma}^2(\omega)}{m^2}}\right], \tag{18}$$

where the cyclotron frequency $\omega_\mathrm{c} = \frac{eB}{mc}$. For the Ohmic dissipation case the susceptibility has four poles at

$$\begin{aligned} \omega = \pm\tilde{\omega}_+ &= \left[\frac{\omega_\mathrm{c} + \mathrm{i}\gamma}{2} \pm \frac{\sqrt{4\omega_0^2 + \omega_\mathrm{c}^2 - \gamma^2 + 2\mathrm{i}\omega_\mathrm{c}\gamma}}{2}\right] \\ \omega = \pm\tilde{\omega}_- &= \left[\frac{-\omega_\mathrm{c} + \mathrm{i}\gamma}{2} \pm \frac{\sqrt{4\omega_0^2 + \omega_\mathrm{c}^2 - \gamma^2 - 2\mathrm{i}\omega_\mathrm{c}\gamma}}{2}\right]. \end{aligned} \tag{19}$$

On the other hand, for the non-Ohmic case these poles cannot be evaluated analytically. The numerical results for the Ohmic dissipation as well as the non-Ohmic dissipation cases are presented below.

In figure 1 we plot the dissipative part of the the x component of susceptibility, i.e. $\chi''_{xx}(\omega)$ versus ω for different values of ω_c and γ, in accordance with equation (17). We note that $\chi''_{xx}(\omega)$ is odd in ω for the Ohmic dissipation case and has Lorentzian line shapes for finite damping values, with peaks centred at the poles. For the non-Ohmic case, $\chi''_{xx}(\omega)$ is even in ω. It is evident from figure 1(b) that, for finite but weak damping, one can obtain all four peaks for the Ohmic dissipation case, whereas for high damping only two peaks are obtained. The same is true for the non-Ohmic case (figure 1(d)). The only difference is that the magnitude of the peak height is higher for the non-Ohmic case and is always positive. Also, the peak width increases with an increase in γ for both the Ohmic and non-Ohmic cases. On the other hand, the width of the peak decreases with an increase in ω_c, as is expected on physical grounds. In the non-Ohmic case, the number of peaks also increases from two to four with an increase in ω_c, whereas it remains two for the Ohmic case with an increase in ω_c, if γ is kept large. Thus, dissipative effects are stronger for the Ohmic case.

In figure 2 we plot the reactive or the real part of the x component of susceptibility ($\chi'_{xx}(\omega)$) versus ω for different values of ω_c and γ, in accordance with equation (17). $\chi'_{xx}(\omega)$ is odd in ω for the Ohmic as well as the non-Ohmic cases. The spreading of the peaks increases, but the peak height decreases with a decrease in ω_c for the Ohmic case. On the other hand, both the spreading and peak height decrease with a decrease in ω_c for the non-Ohmic case. But the features are the same with the variation in γ for both the Ohmic and non-Ohmic cases—the peak height increases but the spreading decreases with an decrease in γ. In addition, the number of peaks increases from one to two with a decrease in γ in the Ohmic as well as non-Ohmic cases.

The z component of the susceptibility tensor is, of course, the same as that of a damped harmonic oscillator, because it has no relation to the cyclotron frequency ω_c.

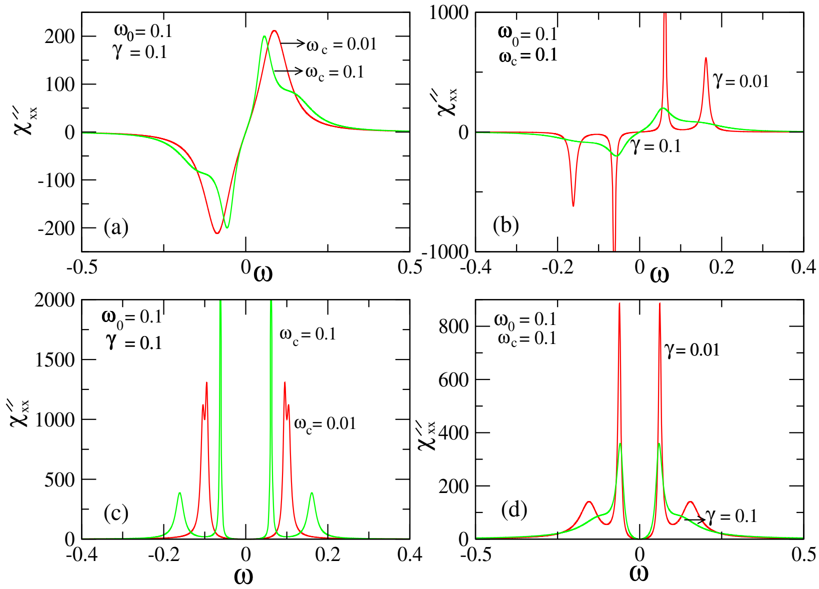

Figure 1. The imaginary part of susceptibility χ_{xx}: (a) Ohmic dissipation case ($J(\omega) \sim \omega$) for two ω_c values; (b) Ohmic dissipation case for two γ values; (c) non-Ohmic dissipation case ($J(\omega) \sim \omega^3$) for two ω_c values; (d) non-Ohmic dissipation case for two γ values.

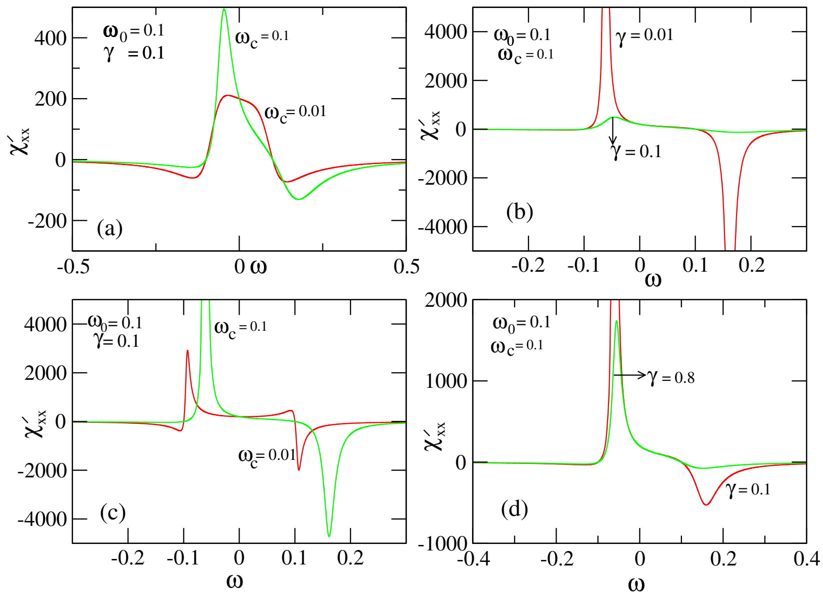

Figure 2. The real part of susceptibility χ_{xx}: (a) Ohmic dissipation case ($J(\omega) \sim \omega$) for two ω_c values; (b) Ohmic dissipation case for two γ values; (c) non-Ohmic dissipation case ($J(\omega) \sim \omega^3$) for two ω_c values; (d) non-Ohmic dissipation case for two γ values.

4. Fluctuation–dissipation relationship: Gibbs approach

In section 3 we calculated the susceptibility as the asymptotic (i.e. $t \to \infty$) response from a fully time-dependent formulation of the underlying QLE. Because detailed balance relations (namely equations (3) and (4)) are built-in within the QLE, as the heat bath is assumed to be in thermal equilibrium at a fixed temperature T, the asymptotic response is expected to be related to the equilibrium properties of the system. This expectation is at the heart of what Kadanoff calls the Einstein approach to statistical mechanics [18] in which equilibrium answers are sought from the asymptotic limit of time-dependent results. It is then natural to ask whether the response obtained from the Einstein approach can be related to spontaneous or equilibrium fluctuations, which can be independently calculated from the standard Gibbsian formulation of equilibrium statistical mechanics. If we can establish this relation, it will not only be tantamount to establishing the fluctuation–dissipation theorem for the phenomenon at hand, but also to demonstrating the equivalence of the Einstein and the Gibbs approaches to statistical mechanics [33].

With this preamble, the position autocorrelation function in equilibrium is defined as

$$C(t) = \langle \vec{x}(t)\cdot\vec{x}(0)\rangle = \mathrm{Tr}\,(\vec{x}(t)\cdot\vec{x}(0)\rho_\beta), \tag{20}$$

where ρ_β is the equilibrium density matrix of the full system and $\vec{x}$ is the two-dimensional position vector in the x–y plane. We determine $C(t)$ by first calculating its imaginary time version, starting from the Euclidean action of the system as described by equation (1):

$$S^{\mathrm{E}}[\vec{x}] = \int_0^{\hbar\beta} \mathrm{d}\tau \left(\frac{m}{2}\dot{\vec{x}}^2 + \frac{m}{2}\omega_0^2\vec{x}^2 + \mathrm{i}m\omega_{\mathrm{c}}(\dot{\vec{x}}\times\vec{x})_z\right) + \frac{1}{2m}\int_0^{\hbar\beta}\mathrm{d}\tau\int_0^{\hbar\beta}\mathrm{d}\sigma\,\tilde{\gamma}(\tau-\sigma)\vec{x}(\tau)\cdot\vec{x}(\sigma) + \int_0^{\hbar\beta}\mathrm{d}\tau\,\vec{f}(\tau)\cdot\vec{x}(\tau), \tag{21}$$

where the first term (within round brackets) takes care of the system part, the second term accounts for the coupling to the environment, and the third term corresponds to the interaction with an external force, in imaginary time. This helps us to determine the correlation function by variation with respect to this force [34, 35]:

$$\langle\vec{x}(\tau)\cdot\vec{x}(\sigma)\rangle = \hbar^2\mathrm{Tr}\left(\frac{\delta}{\delta\vec{f}(\tau)}\frac{\delta}{\delta\vec{f}(\sigma)}\rho_\beta\right)_{\vec{f}=0}. \tag{22}$$

It is sufficient to restrict ourselves to the classical path for the calculation of the autocorrelation function [34, 35]. Thus the Fourier representation of the classical Euclidean action becomes [33, 34]

$$S^{\mathrm{E}}_{\mathrm{cl}} = -\frac{1}{2m\hbar\beta}\sum_{n=-\infty}^{+\infty}\left[\frac{1}{\nu_n^2+\frac{\tilde{\gamma}(|\nu_n|)\nu_n}{m}+\omega_0^2-\mathrm{i}\omega_{\mathrm{c}}\nu_n}+\frac{1}{\nu_n^2+\frac{\tilde{\gamma}(|\nu_n|)\nu_n}{m}+\omega_0^2+\mathrm{i}\omega_{\mathrm{c}}\nu_n}\right] \times\int_0^{\hbar\beta}\mathrm{d}\tau\int_0^{\hbar\beta}\mathrm{d}\sigma\,\vec{f}(\tau)\vec{f}(\sigma)\exp(\mathrm{i}\nu_n(\tau-\sigma)), \tag{23}$$

where $\nu_n = \frac{2\pi n}{\hbar\beta}$ are the so-called Matsubara frequencies. Since the force appears only through the action in the exponent of the equilibrium density matrix, we can easily evaluate the functional derivatives according to equation (21) and obtain the position autocorrelation function in imaginary time:

$$C(\tau) = \frac{1}{m\beta}\sum_{n=-\infty}^{+\infty}\left[\frac{1}{\nu_n^2+\frac{\tilde{\gamma}(|\nu_n|)\nu_n}{m}+\omega_0^2-\mathrm{i}\omega_{\mathrm{c}}\nu_n}+\frac{1}{\nu_n^2+\frac{\tilde{\gamma}(|\nu_n|)\nu_n}{m}+\omega_0^2+\mathrm{i}\omega_{\mathrm{c}}\nu_n}\right]\exp(\mathrm{i}\nu_n\tau). \tag{24}$$

The real-time correlation function cannot be obtained by simply replacing τ by it, because for negative times the sum does not converge. The idea is to express the sum in equation (23) as a contour integral in the complex frequency plane and look for a function which is well

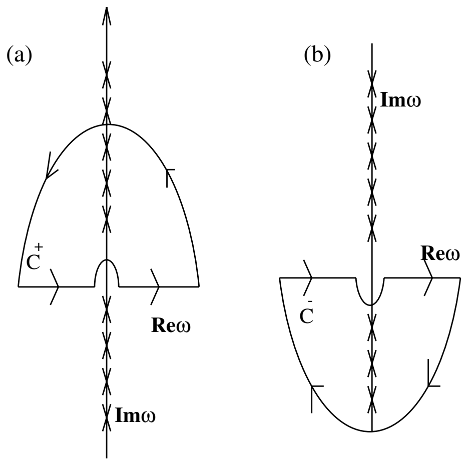

Figure 3. The analytic continuation of the imaginary time correlation function to real times by using the contours depicted in (a) and (b) to obtain equations (24) and (25) respectively.

behaved at infinity, but has poles at $\omega = \mathrm{i}\nu_n$ [36]. This requirement is fulfilled by the term: $\frac{\hbar\beta}{1-\exp(-\hbar\beta\omega)}$. Now, doing the integration along the contour shown in figures 3(a) and (b) and after some algebra, we find the real-time correlation function as

$$C(t) = \frac{\hbar}{\pi m^2}\int_{-\infty}^{+\infty} \mathrm{d}\omega \left[\frac{\tilde{\gamma}(\omega)\omega}{(\omega^2-\omega_0^2-\omega\omega_{\mathrm{c}})^2+\frac{\tilde{\gamma}^2(\omega)\omega^2}{m^2}} + \frac{\tilde{\gamma}(\omega)\omega}{(\omega^2-\omega_0^2+\omega\omega_{\mathrm{c}})^2+\frac{\tilde{\gamma}^2(\omega)\omega^2}{m^2}}\right] \times \frac{\mathrm{e}^{(-\mathrm{i}\omega t)}}{1-\mathrm{e}^{(-\hbar\beta\omega)}}. \qquad (25)$$

It is easy to show from equation (24) that

$$\tilde{C}(\omega) = \frac{2\hbar}{1-\exp(-\beta\hbar\omega)}\chi''_{xx}(\omega). \qquad (26)$$

Equation (26) represents the fluctuation–dissipation theorem in the context of dissipative Landau diamagnetism. The position autocorrelation function describes the spontaneous fluctuations of the system, while the imaginary part of the dynamic susceptibility χ''_{xx} determines the energy dissipation in the system due to work done by an external weak force.

5. Coherence–decoherence transition

In this section our discussion is focused on the destruction of quantum coherence by environment-induced dissipation in the context of Landau diamagnetism. Two questions are relevant: (i) can we quantify the criterion for crossover from coherent to decoherent dynamics?; (ii) Is this criterion universal? As far as some model systems are concerned, the answer to (i) is in the affirmative [25]. Regarding the question (ii), there seems to be no universality in the criterion of crossover. As a matter of fact, the value of the crossover parameter depends on the particular quantity under consideration and its initial preparation. Thus, quantum memory effects play a crucial role as the system makes a transition from the coherent to the decoherent regime. To clarify this issue, we focus on dissipative diamagnetism and consider its $T = 0$ behaviour, wherein quantum coherence is the most prominent. Here we follow the discussion of Egger *et al* [25].

We start with the QLE for dissipative Landau diamagnetism subject to Ohmic damping. The motion in the x–y plane can be expressed in the compact form:

$$\ddot{Z} + \bar{\gamma}\dot{Z} + \omega_0^2 Z = \frac{\theta(t)}{m}, \tag{27}$$

where $Z = x + \mathrm{i}y$, $\bar{\gamma} = \gamma + \mathrm{i}\omega_c$, and $\theta = F_x + \mathrm{i}F_y$. Thus, the time dependence of the corresponding classical quantity (*a la* Ehrenfest) is governed by the following equation:

$$\langle\ddot{Z}\rangle + \bar{\gamma}\langle\dot{Z}\rangle + \omega_0^2\langle Z\rangle = \frac{\theta(t)}{m}, \tag{28}$$

where the angular brackets represent statistical averages over the ground-state properties ($T = 0$), i.e. the expectation values. As discussed earlier, the response to an external force is characterized by the generalized susceptibility $\chi_{\mathrm{osc}}(t)$ [32]:

$$\langle Z(t)\rangle = \frac{1}{m\omega_0}\int_{-\infty}^{t} \mathrm{d}t'\,\chi_{\mathrm{osc}}(t - t')\theta(t'). \tag{29}$$

From equations (27) and (28), we obtain the Fourier transform of $\chi_{\mathrm{osc}}(t)$ as

$$\chi_{\mathrm{osc}}(\omega) = \frac{\omega_0}{\omega_0^2 - \omega^2 - \mathrm{i}\bar{\gamma}\omega}. \tag{30}$$

On the other hand, using the fluctuation–dissipation theorem [32], $\chi_{\mathrm{osc}}(\omega)$ can be related to the spectral function $S_{\mathrm{osc}}(\omega)$, which in turn determines the equilibrium correlation function $C_{\mathrm{osc}}(\omega)$. The functional relationship which holds at $T = 0$ is as follows:

$$\mathrm{Im}\,\chi_{\mathrm{osc}}(\omega) = \omega S_{\mathrm{osc}}(\omega) = \frac{\omega}{|\omega|}C_{\mathrm{osc}}(\omega), \tag{31}$$

where $C_{\mathrm{osc}}(t) = \mathrm{Re}\langle Z(t)Z(0)\rangle$. Using equations (29) and (30), we obtain the spectral function

$$S_{\mathrm{osc}}(\omega) = \frac{\gamma\omega_0}{(\omega_0^2 - \omega^2 + \omega\omega_c)^2 + \gamma^2\omega^2}. \tag{32}$$

The quantity $S_{\mathrm{osc}}(\omega)$ can be used as a signature for the transition from coherence to decoherence: $S_{\mathrm{osc}}(\omega)$ has two inelastic peaks at $\omega_m = \frac{\omega_0}{2}\left[-\kappa_2 \pm \sqrt{4 - \kappa_1^2 + \kappa_2^2}\right]$ for weak damping, where κ_1 and κ_2 are dimensionless parameters defined by $\kappa_1 = \frac{\gamma}{\omega_0}$ and $\kappa_2 = \frac{\omega_c}{\omega_0}$. These two quantities are employed as the crossover parameters to quantify the coherence-to-decoherence transition. Defining $\bar{\kappa}^2 = \kappa_1^2 + \kappa_2^2$, we can say that, below the critical coherent criterion (defined below, cf equation (34)), i.e. $\bar{\kappa}^2 < \bar{\kappa}_c^2$, the function $S_{\mathrm{osc}}(\omega)$ exhibits two inelastic peaks which are evident from figure 4, in which we plot $S_{\mathrm{osc}}(\omega)$ versus ω for different κ_1 and κ_2. At the critical coherent criterion (cf equation (34)) the two peaks merge into a single quasi-elastic peak. The latter persists for $\bar{\kappa}^2 > \bar{\kappa}_c^2$. Since the quasi-elastic peak is centred near $\omega \simeq 0$, we can make a small-ω expansion of $S_{\mathrm{osc}}(\omega)$:

$$S_{\mathrm{osc}}(\omega) \simeq \kappa_1\chi_0^2\left[1 - \kappa_2\chi_0\omega + (2 - \kappa_1^2 - \kappa_2^2)\chi_0^2\omega^2 + \mathrm{O}(\omega^3)\right], \tag{33}$$

where $\chi_0 = \frac{1}{\omega_0}$. The critical line is determined by inspecting the sign of the curvature of $S_{\mathrm{osc}}(\omega)$. The latter is positive (implying coherence) if $\frac{\mathrm{d}^2 S_{\mathrm{osc}}(\omega)}{\mathrm{d}\omega^2} > 0$, or

$$\bar{\kappa}^2 = \kappa_1^2 + \kappa_2^2 < 2. \tag{34}$$

But the curvature changes sign at the critical line:

$$\bar{\kappa}_c^2 = \kappa_1^2 + \kappa_2^2 = 2, \tag{35}$$

and hence the system goes to the decoherent region when

$$\bar{\kappa}^2 = \kappa_1^2 + \kappa_2^2 > 2. \tag{36}$$

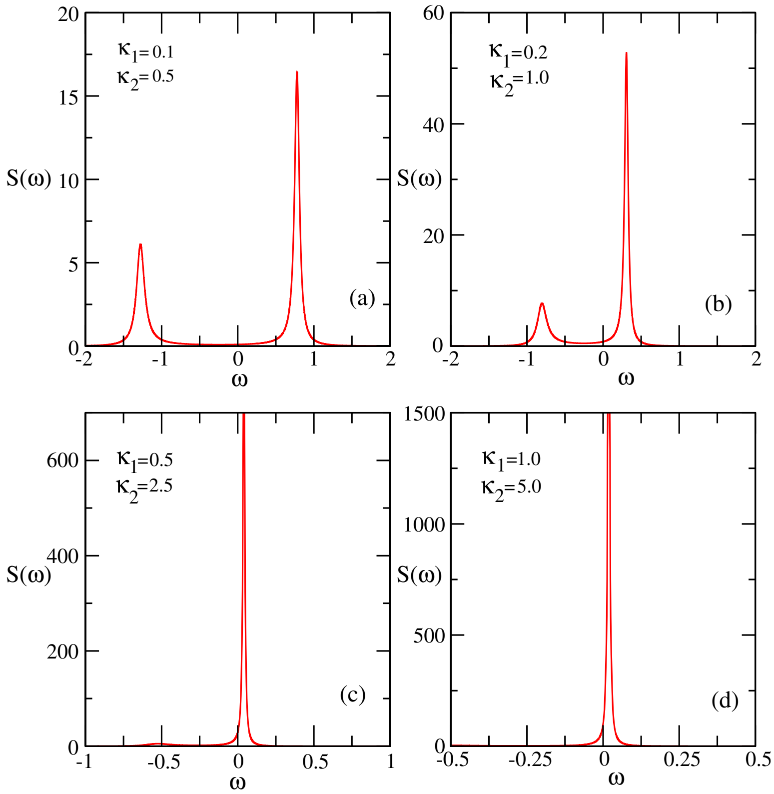

Figure 4. Spectral function $S_{osc}(\omega)$ versus ω with Ohmic dissipation for dissipative Landau diamagnetism for different parameter values.

It is illustrative to compare this behaviour with that of the damped harmonic oscillator which was discussed by Egger *et al* [25]. From figures (4) and (5) one notes that, for the damped oscillator case, $S_{osc}(\omega)$ has two inelastic peaks of equal height for weak damping. As the one-parameter damping strength increases, these two peaks approach each other and, at the critical damping strength (α_c), the two peaks merge into a single quasi-elastic peak at $\omega = 0$ which persists for $\alpha > \alpha_c$. On the other hand, for dissipative diamagnetism, the coherent–decoherent transition is to be examined in a two-parameter plane, defined by κ_1 and κ_2. One obtains two inelastic peaks which are not of equal height for low values of κ_1 and κ_2, because the peaks are not symmetric on either side of $\omega = 0$. As one increases κ_1 and κ_2, the peak height of the small peak decreases and eventually vanishes at the critical line to yield a single peak which is not at $\omega = 0$, but near $\omega = 0$. Above the critical line, the single quasi-elastic peak persists.

We turn next to a different criterion for quantifying the transition from coherence to decoherence, which is based on the quantity $P_{osc}(t)$, defined as follows:

$$P_{osc}(t) = \frac{\langle Z(t)\rangle}{Z_0}. \tag{37}$$

We are interested in the relaxation of the expectation value $\langle Z(t)\rangle$ starting from a non-equilibrium initial state. Applying the force $F(t) = m\omega_0^2 Z_0$ for $t < 0$, the initial condition $\langle Z(0)\rangle = Z_0$ is prepared and the corresponding dynamical quantity $P_{osc}(t)$ is computed, after switching off the force $F(t)$, at $t = 0$. Following the damped quantum harmonic oscillator

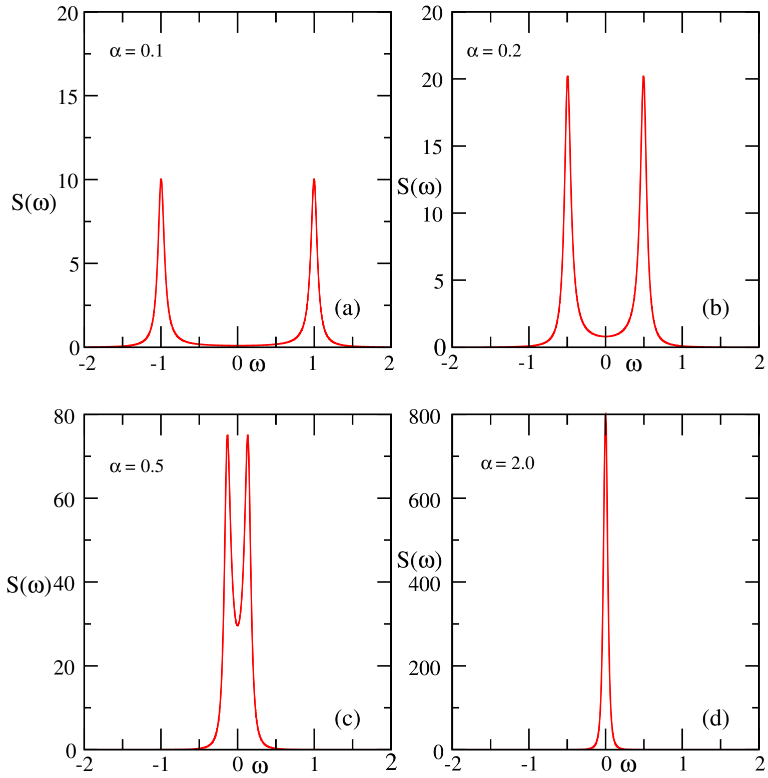

Figure 5. Spectral function $S_{osc}(\omega)$ versus ω with Ohmic dissipation for a damped harmonic oscilltor for different parameter values.

case [25], we may now write

$$P_{osc}(t) = \mathrm{Re}\left[\frac{\cos(\bar{\Omega}t - \bar{\phi})\exp(-\frac{\bar{\gamma}t}{2})}{\cos(\bar{\phi})}\right], \tag{38}$$

where

$$\begin{aligned}\bar{\Omega} &= \sqrt{\omega_0^2 - \frac{\bar{\gamma}^2}{4}} = \Omega' + \mathrm{i}\Omega''\\ \bar{\phi} &= \mathrm{Re}\left[\tan^{-1}\left(\frac{\bar{\gamma}}{2\bar{\Omega}}\right)\right].\end{aligned} \tag{39}$$

Defining $a = (\omega_0^2 + \frac{\omega_c^2}{4} - \frac{\gamma^2}{4})$ and $b = \frac{\gamma\omega_c}{2}$,

$$\begin{aligned}\Omega' &= \frac{1}{\sqrt{2}}\sqrt{a + \sqrt{a^2 + b^2}},\\ \Omega'' &= \frac{1}{\sqrt{2}}\sqrt{\sqrt{a^2 + b^2} - a},\\ \bar{\phi} &= \tan^{-1}(X),\\ X &= \frac{\gamma\Omega' + \Omega''\omega_c}{2(\Omega'^2 + \Omega''\omega_c)},\end{aligned} \tag{40}$$

we find

$$P_{\rm osc}(t) = \left[\frac{\cos(\Omega' t - \bar{\phi})\cos(\Omega'' t)\cos(\frac{\omega_c t}{2}) - \sin(\Omega' t - \bar{\phi})\sin(\Omega'' t)\sin(\frac{\omega_c t}{2})}{\cos(\bar{\phi})}\right]\exp\left(-\frac{\gamma t}{2}\right). \tag{41}$$

The signature of coherence is now damped-oscillatory behaviour if $b^2 > 0$ and $a^2 + b^2 > 0$. Thus the important inequality condition for the system to be coherent is:

$$(1 - \kappa_1^2 + \kappa_2^2)^2 + \frac{(\kappa_1\kappa_2)^2}{4} > 0. \tag{42}$$

The system crosses over to relaxational (decoherent) behaviour at the critical line

$$\left(1 - \frac{\kappa_1^2}{4} + \frac{\kappa_2^2}{4}\right)^2 + \frac{(\kappa_1\kappa_2)^2}{4} = 0, \tag{43}$$

which is clearly different from the criterion mentioned above (cf equation (34)). Thus the criterion for crossover from coherence to decoherence depends on the specific physical quantity considered. This conclusion is identical to the cases of damped quantum harmonic oscillator as well as the spin-Boson model [25].

6. Summary and conclusions

Here we have analysed an exact treatment of the Feynman–Vernon model of a charged Brownian particle in a magnetic field in the quantum dissipative regime. Starting from the QLE, we have derived the generalized susceptibility tensor, and have discussed its real and imaginary parts for the particular case when the magnetic field $\vec{B}$ is along the z-axis. Following the Gibbs ensemble approach, we have calculated the position autocorrelation function that measures the spontaneous fluctuations of the system degrees of freedom due to coupling with the environment. The latter has been shown to be related to the imaginary part of the susceptibility that measures the energy dissipation of the system due to irreversible energy transfer between the system and the environment. The aforesaid treatment then exemplifies the fluctuation–dissipation theorem in the context of dissipative diamagnetism as well as establishes the equivalence of the Einstein and the Gibbs approaches to statistical mechanics for the case at hand. Environment-induced decoherence is an important issue in mesoscopic systems and quantum information processes. We have discussed this in the context of dissipative diamagnetism and have argued that the transition from the Landau to the Bohr-Van Leeuwen regime can indeed be viewed as a coherence-to-decoherence transition. Further, it has been demonstrated that the initial preparation of a dissipative quantum system leads to abrupt changes regarding the criterion for coherent-to-decoherent transition. As in glassy systems characterized by hysteretic behaviour, quantum systems also exhibit memory of their initial state of preparation.

In conclusion, we have presented a unified treatment of threefold response, i.e. fluctuation, dissipation and decoherence of a system, due to its coupling with the environment in the context of the contemporarily important topic of dissipative diamagnetism. We have established the equivalence of the equilibrium and non-equilibrium statistical physics for a phenomena like Landau–Drude diamagnetism, which is inherently quantum and strongly dependent on boundary effects. Finally, we have demonstrated that the coherent-to-incoherent transition depends to a large degree on the particular dynamical quantity under consideration (e.g. correlation function, occupation probability, etc), as well as initial conditions of preparation. Our derived results should be of some interest in the presently active area of mesoscopic structures.

Acknowledgments

MB acknowledges financial support from the Council of Scientific and Industrial Research (CSIR), Government of India.

References

[1] Sakurai J J 1985 *Modern Quantum Mechanics* (Reading, MA: Addison-Wesley)
[2] Landau L 1930 *Z. Phys.* **64** 629
[3] Van Vleck J H 1932 *The Theory of Electric and Magnetic Susceptibilities* (London: Oxford University Press)
[4] Imry Y 2002 *Introduction to Mesoscopic Physics* (London: Oxford University Press) (*Preprint* cond-mat/0202044v1)
[5] Hong T M and Wheatley J M 1991 *Phys. Rev.* B **43** 5702
Hong T M and Wheatley J M 1990 *Phys. Rev.* B **42** 6492
[6] See, for instance Ashcroft N and Mermin D 1976 *Solid State Physics* (New York: Holt, Rinehart and Winston)
[7] Caldeira A O and Leggett A J 1984 *Ann. Phys.* **149** 374
[8] Caldeira A O and Leggett A J 1993 *Physica* A **121** 587
[9] Leggett A J, Chakravarty S, Dorsey A T, Fisher M P A, Garg A and Zwerger W 1987 *Rev. Mod. Phys.* **59** 1
[10] Weiss U 1999 *Quantum Dissipative Systems* (Singapore: World Scientific)
[11] Dittrich T, Hänggi P, Ingold G L, Kramer B, Schón G and Zwerger W 1998 *Quantum Transport and Dissipation* (New York: Wiley–VCH)
[12] See for instance Zurek W H 1991 *Phys. Today* **44** 36
[13] Dattagupta S and Singh J 1997 *Phys. Rev. Lett.* **79** 961
[14] Ford G W, Kac M and Mazur P 1965 *J. Math. Phys.* **6** 504
Ford G W, Lewis J T and O'Connell R F 1988 *Phys. Rev.* A **37** 4419
[15] Li X L, Ford G W and O'Connell R F 1990 *Phys. Rev.* A **41** 5287
Li X L, Ford G W and O'Connell R F 1990 *Phys. Rev.* A **42** 4519
Li X L, Ford G W and O'Connell R F 1996 *Phys. Rev.* E **53** 3359
[16] Zwanzig R 1973 *J. Stat. Phys.* **1** 215
[17] Feynman R P 1948 *Rev. Mod. Phys.* **20** 367
[18] Kadanoff L P 2000 *Statistical Physics—Statics, Dynamics and Renormalization* (Singapore: World Scientific)
[19] Ford G W, Lewis J T and O'Connell R F 1985 *Phys. Rev. Lett.* **55** 2273
[20] Garraway B M and Suominen K A 1995 *Rep. Prog. Phys.* **58** 365
[21] Grabert H and Wipf H 1990 *Advances in Solid State Physics* vol 30 (Braunschweig: Vieweg)
[22] Chandler D 1991 *Liquids, Freezing and the Glass Transition* (*Les Houches Lectures*) ed D Levesque *et al* (Amsterdam: Elsevier Science)
[23] Bohr N 1911 *PhD Thesis* University of Copenhagen
[24] Van Leeuwen J H 1921 *J. Physique* **2** 361
[25] Egger R, Grabert H and Weiss U 1997 *Phys. Rev.* E **55** R3809
See also Dattagupta S and Puri S 2004 *Dissipative Phenomena in Condensed Matter Physics* (Berlin: Springer)
[26] Darwin C G 1930 *Proc. Camb. Phil. Soc.* **27** 86
[27] Zwanzig R 1960 *J. Chem. Phys.* **33** 1338
Zwanzig R 1961 *Lectures in Theoretical Physics (Boulder)* vol 3, ed W E Brittin, B W Downs and J Down (New York: Interscience)
[28] Prigogine J and Resibois P 1961 *Physica* **27** 629
[29] Chang L D and Chakravarty S 1985 *Phys. Rev.* B **31** 154
[30] Louis A A and Sethna J P 1995 *Phys. Rev. Lett.* **74** 1363
[31] Grabert H and Schober H R 1994 *Hydrogen in Metals* vol II, ed H Wipf (Heidelberg: Springer)
[32] See, for instance Dattagupta S 1987 *Relaxation Phenomena in Condensed Matter Physics* (Orlando, FL: Academic)
[33] Bandyopadhyay M and Dattagupta S 2006 *J. Stat. Phys.* **123** 1273
[34] Ingold G L 2002 *Springer Lecture Notes in Physics* vol 611 (Berlin: Springer) pp 1–53
[35] Grabert H, Weiss U and Talkner P 1984 *Z. Phys.* B **55** 87
[36] Reif F 1965 *Statistical Physics, Berkley Physics Course* vol 5 (New York: McGraw-Hill)